

the zeroes and the coefficients are two views of one thing

What counts as a polynomial *

- an expression $p(x) = a_n x^n + \dots + a_1 x + a_0$ in which every power of x is a whole number
- the degree is the highest power of x that actually shows up
- degree 1 is linear, degree 2 is quadratic, degree 3 is cubic
- $\sqrt{x}$ and $\frac{1}{x}$ are shut out: their powers are $\frac{1}{2}$ and -1 , not whole numbers

Value at a point, and a zero

the value of p at $x=k$

$$p(k)$$

k is a zero of p exactly when

$$p(k) = 0$$

zero of the linear $ax+b$

$$x = -\frac{b}{a}$$

check on $2x-6$

$$p(3) = 2(3) - 6 = 0$$

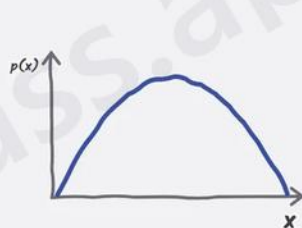
a zero is a value of x , never a value of $p(x)$. $p(x)$ is the thing that becomes zero there

Named by degree

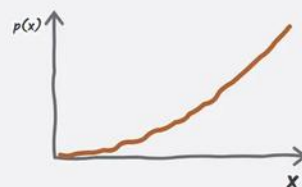
degree	general form	zeroes
0, constant	$p(x) = 5$	none at all
1, linear	$ax+b, a \neq 0$	exactly one
2, quadratic	ax^2+bx+c	at most two
3, cubic	ax^3+bx^2+cx+d	at most three

the zero polynomial $p(x) = 0$ is the odd one out: its degree is not defined, and every x is a zero of it

Geometrical meaning of the zeroes



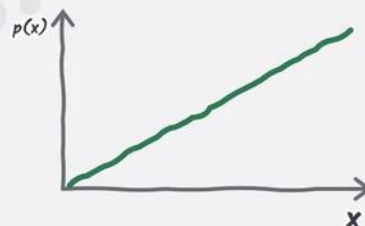
a curve rising off the axis and coming back down to it: it meets the axis at two separate points, so two distinct zeroes



a curve running flat along the axis on the left and then climbing steadily away: the one place it leaves the axis is a repeated zero

a zero is exactly a place where the graph cuts or touches the x axis. Count those places and you have counted the real zeroes

One zero, and none at all *



a straight line for a linear polynomial: it leaves the axis at one point and never returns, so exactly one zero

no real zeroes has nothing to draw on the axis at all. Lift a parabola clear of the x axis, wholly above it or wholly below it, and the curve never meets the axis, so that quadratic has none. A polynomial of degree n has at most n