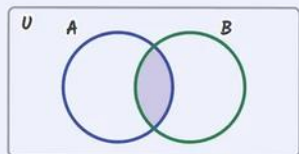
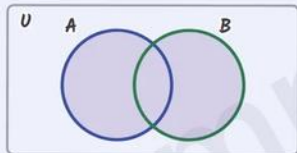


once one event is known, the sample space itself shrinks down to it

Reading two events on one diagram



$P(A \text{ and } B)$: the lens where both circles overlap, exactly the numerator conditional probability divides by $P(B)$



$P(A \text{ or } B)$: the combined area of both events, counted once even where they overlap

Independent vs mutually exclusive

Independent events

- $P(A \cap B) = P(A) P(B)$
- knowing one happened changes nothing about the other's chance

Mutually exclusive events

- A and B share no outcomes, so $P(A \cap B) = 0$
- if both have nonzero probability, knowing one happened rules the other out completely

Conditional probability *

probability of A, given B has already happened

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

defined only when

$$P(B) > 0$$

the symmetric version

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, P(A) > 0$$

conditioning on B re-scales the whole sample space down to B: everything outside B stops counting at all

the two ideas get mixed up often, but a mutually exclusive pair with nonzero probabilities is actually the most dependent kind of pair there is

independence is a **PRODUCT** test, $P(A \text{ and } B) = P(A)P(B)$. Conditional probability **RE-SCALES** the sample space down to B. Keep the two ideas separate even though both involve A and B together

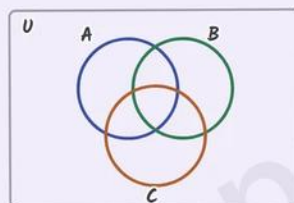
Multiplication theorem

for two events $P(A \cap B) = P(A) P(B|A) = P(B) P(A|B)$

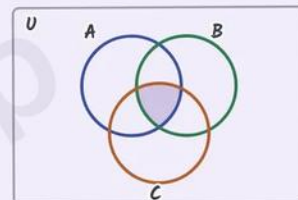
extended

to three events $P(A \cap B \cap C) = P(A) P(B|A) P(C|A \cap B)$

Three events sharing one sample space *



three events drawn in the same sample space, how their circles overlap is exactly what a compound probability question is asking about



the very centre where all three circles overlap at once: the region $P(A \text{ and } B \text{ and } C)$