

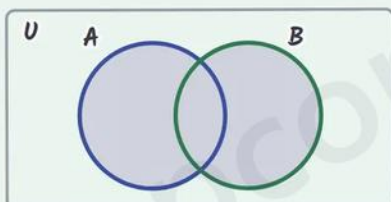
events are just sets, and every probability rule is set theory wearing a disguise

Basic terminology

- a random experiment has more than one known outcome, but the actual result can't be predicted
- the sample space S (or Ω) is the set of ALL possible outcomes. $n(S)$ counts them
- an event is any subset of the sample space; a favourable outcome belongs to that subset
- elementary event: one outcome. certain event: all of S . impossible event: the empty set $\varnothing$

Algebra of events

- $A \cup B$: outcomes belonging to at least one of A, B
- $A \cap B$: outcomes belonging to both A and B
- A and B are exhaustive if $A \cup B = S$
- A and B are mutually exclusive (disjoint) if $A \cap B = \varnothing$, no outcome in common
- A, B are complementary if BOTH mutually exclusive AND exhaustive: $A \cup B = S, A \cap B = \varnothing$

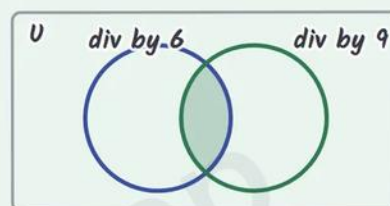


$A \cup B$, shaded as one region at one shade: every outcome in A or in B or in both

Union and intersection from a definition

Q. S is the positive integers up to 50. A is the subset divisible by 6, B the subset divisible by 9. Find $A \cup B$ and $A \cap B$

- $A = \{6, 12, 18, 24, 30, 36, 42, 48\}$, $B = \{9, 18, 27, 36, 45\}$
- $A \cup B$ collects every element of either set
- $A \cap B$ keeps only what both share: 18 and 36, the multiples of $\text{lcm}(6, 9) = 18$



A (divisible by 6) and B (divisible by 9): the shaded overlap is $A \cap B = \{18, 36\}$

Answer

$A \cup B = \{6, 9, 12, 18, 24, 27, 30, 36, 42, 45, 48\}$,
 $A \cap B = \{18, 36\}$

Classical probability

definition, all outcomes equally likely $P(A) = \frac{n(A)}{n(S)}$

$n(A)$ is the count of favourable outcomes, $n(S)$ the count of every possible outcome

Reading combinations of events *

Operation	Interpretation
A'	not A
$A \cup B$	at least one of A, B
$A \cap B$	both A and B
$(A' \cap B) \cup (A \cap B')$	exactly one of A, B
$(A \cup B)'$	neither A nor B

$(A \cup B)' = A' \cap B'$, by De Morgan's law: not- A -or- B is the same event as not- A -and-not- B

Elementary properties

Complement

$$P(A') = 1 - P(A)$$

Bounds

$$0 \leq P(A) \leq 1$$

Impossible, certain

$$P(\varnothing) = 0, P(S) = 1$$

Mutually exclusive

$$P(A \cup B) = P(A) + P(B)$$

Subset

$$A \subset B \Rightarrow P(A) \leq P(B)$$

Set difference

$$P(A \cap B') = P(A) - P(A \cap B)$$