

Propositional Logic and Hardware Implementation

from a sentence that is true or false, to the gates that add two numbers



A proposition is a sentence with a truth value

- a **propositional variable** stands for a statement that is either true or false, never both and never neither
- connectives build bigger statements: NOT for negation, AND for conjunction, OR for disjunction, then implication and equivalence
- a **well formed formula** is one built by the rules: variables are wff, and if p and q are wff then so are NOT p , p AND q , p OR q and the rest, with brackets making the grouping explicit
- an **interpretation** assigns true or false to every variable, and the formula then has one truth value under it

The five connectives

NOT p

negation, true exactly when p is false

p AND q

conjunction, true only when both are true

p OR q

disjunction, true when at least one is true, so it is the inclusive or

p implies q

false only when p is true and q is false, true in every other case including when p is false

p equivalent q

true when both sides carry the same truth value

p	q	p AND q	p	q	p OR q	p	q	p to q
0	0	0	0	0	0	0	0	1
0	1	0	0	1	1	0	1	1
1	0	0	1	0	1	1	0	0
1	1	1	1	1	1	1	1	1

inclusive

implication: false on exactly one row, true p with false q

conjunction: one row true out of four

disjunction: only the all false row is false

read the output column downward, that column IS the formula's meaning

