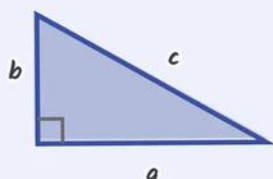


the right angle fixes the relation between the three sides, and the converse tests for it

The theorem and its converse

- **theorem**: in a right angled triangle, the square on the hypotenuse equals the sum of the squares on the other two sides
- the **hypotenuse** is the side opposite the right angle, and it is always the longest of the three
- **converse**: if in a triangle the square of one side equals the sum of the squares of the other two, then the angle opposite that first side is a right angle
- the theorem starts from a right angle and gives a relation. The converse starts from the relation and **proves** the right angle
- which one a question needs is decided by what is given: an angle given means the theorem, three lengths given means the converse



a right angled triangle with the two shorter sides marked a and b and the hypotenuse c drawn opposite the right angle: the relation $a^2 + b^2 = c^2$ belongs to this arrangement and no other

The relations worth knowing by heart

the theorem itself, c being the hypotenuse

$$a^2 + b^2 = c^2$$

finding a shorter side instead

$$a = \sqrt{c^2 - b^2}$$

the altitude on the hypotenuse, from similarity

$$p^2 = xy$$

the diagonal of a rectangle of sides l and b

$$d = \sqrt{l^2 + b^2}$$

the altitude of an equilateral triangle of side a

$$h = \frac{\sqrt{3}}{2} a$$

the diagonal of a square of side a

$$d = a\sqrt{2}$$

the last three are all the theorem applied once and then remembered, which is faster than rebuilding them under time pressure

Pythagorean triplets, and why they save time

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- three whole numbers satisfying $a^2 + b^2 = c^2$ form a **Pythagorean triplet**
- the ones worth recognising on sight: 3, 4, 5 and 5, 12, 13 and 8, 15, 17 and 7, 24, 25
- any **multiple** of a triplet is also a triplet, so 6, 8, 10 and 9, 12, 15 and 15, 20, 25 all come from 3, 4, 5
- spotting a triplet means the answer is available without a square root, which is where the time is saved
- the **largest number** is always the hypotenuse, so in 5, 12, 13 the right angle sits opposite 13