

one unknown, highest power two, and three separate ways to reach the roots

Is it quadratic?

$$ax^2 + bx + c = 0, a \neq 0$$

one variable, highest index exactly two

→ narrow it down

Does it factorise?

$$x^2 - 5x + 6 = (x-2)(x-3)$$

split the middle term, fastest route when it works

→ narrow it down

Otherwise

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

the formula never fails, so it is the fallback

the condition $a \neq 0$ is the whole definition: if a were zero the x^2 term would vanish and the equation would be linear, with one root instead of two

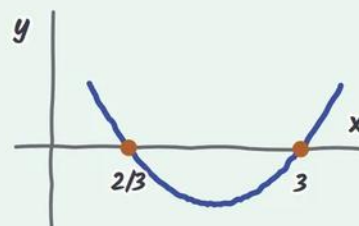
Standard form, and why it comes first

- every method below assumes the equation is written as $ax^2 + bx + c = 0$ with a , b and c real and $a \neq 0$
- collect every term on one side before reading off a , b and c , so $3x^2 = 5x - 2$ must become $3x^2 - 5x + 2 = 0$
- a is the coefficient of x^2 , b of x , and c is the constant, each carried with its own sign
- the sign travels with the number: in $x^2 - 7x + 12 = 0$ the value of b is -7 , not 7
- a value of x that satisfies the equation is called a **root** of the equation, and a quadratic has two of them

Splitting the middle term in full

Q. Solve $3x^2 - 11x + 6 = 0$ by factorisation.

- here $a = 3$, $b = -11$ and $c = 6$, so $ac = 18$
- two numbers with product 18 and sum -11 are -9 and -2 , both negative because the product is positive and the sum negative
- split: $3x^2 - 9x - 2x + 6 = 0$
- group: $3x(x - 3) - 2(x - 3) = 0$
- take out the common bracket: $(x - 3)(3x - 2) = 0$
- so $x - 3 = 0$ or $3x - 2 = 0$, giving $x = 3$ or $x = \frac{2}{3}$



the curve $y = 3x^2 - 11x + 6$ crossing the horizontal axis at $2/3$ and at 3 : the two roots are exactly the two crossings, which is why a quadratic has two of them

Answer $x = 3$ and $x = \frac{2}{3}$

fastest when the numbers are kind

Solving by factorisation *

write the equation in standard form, so one side is exactly zero

multiply a by c and look for two numbers whose product is ac and whose sum is b

split the middle term using those two numbers, turning three terms into four

group the four terms in pairs and take out the common factor from each pair

the bracket left over is common, so the expression becomes a product of two factors

set each factor to zero separately, because a product is zero only when a factor is zero