

comparing two quantities by division, and the operations that keep a proportion true

Ratio, and the rules it obeys

- a **ratio** compares two quantities of the **same kind** by division, written $a : b$ and meaning $\frac{a}{b}$
- the first term a is the **antecedent** and the second b the **consequent**
- the two quantities must be in the **same units** before the ratio is written, so 50 cm to 2 m becomes 50 : 200
- a ratio itself has **no units**, since the units cancel in the division
- multiplying or dividing **both terms by the same non zero number** leaves the ratio unchanged, so 6 : 8 is 3 : 4
- a ratio is normally given in its **lowest terms**, with the two parts having no common factor

Operations that preserve a proportion *

the basic property, cross multiplication $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$

invertendo, turn both sides over $\frac{b}{a} = \frac{d}{c}$

alternendo, swap the means $\frac{a}{c} = \frac{b}{d}$

componendo, add one to each side $\frac{a+b}{b} = \frac{c+d}{d}$

dividendo, subtract one from each side $\frac{a-b}{b} = \frac{c-d}{d}$

componendo and dividendo together $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

the last line is the one worth memorising: it turns a messy equation with a sum over a difference into a simple ratio in one step

Proportion, and the means and extremes

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- four quantities are in **proportion** when $a : b = c : d$, written $a : b :: c : d$
- a and d are the **extremes**, the outer pair, and b and c the **means**, the inner pair
- the defining property is that the **product of the extremes equals the product of the means**, so $ad = bc$
- that cross multiplication is what turns any proportion into an ordinary equation
- three quantities are in **continued proportion** when $a : b = b : c$, and then b is the **mean proportional** between a and c
- in that case $b^2 = ac$, so the mean proportional is $\sqrt{ac}$

$$\left[\begin{array}{cccc} a & b & c & d \end{array} \right]$$

the four terms of a proportion laid out in order: the two on the outside are the extremes and the two in the middle are the means. Multiplying the outer pair and the inner pair gives the same answer, which is the whole content of $ad = bc$