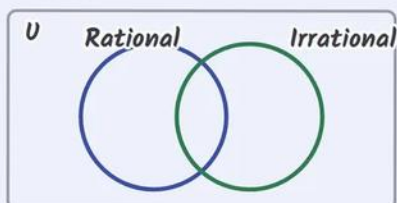


how the number families nest, what makes a decimal rational, and working with surds

The families, from smallest outwards

- natural numbers N are the counting numbers 1, 2, 3 and onwards
- whole numbers W are the naturals together with 0
- integers Z add the negatives, so ... -2, -1, 0, 1, 2 ...
- rational numbers Q are those writable as $\frac{p}{q}$ with p and q integers and $q \neq 0$
- irrational numbers cannot be written that way at all, such as $\sqrt{2}$, $\sqrt{3}$ and π
- real numbers R are the rationals and irrationals together, and every point on the number line is one



the two circles sitting side by side without overlapping, both inside the rectangle of real numbers: no number is both rational and irrational, and together they fill the real line completely

How the families nest

- N is inside W , W inside Z , and Z inside Q , each family adding something the one before lacked
- so every natural number is also an integer and a rational, but not every integer is natural
- every integer is rational, since 5 can be written as $\frac{5}{1}$
- the rationals and the irrationals are completely separate: no number belongs to both
- together they make the reals, and there is no gap left on the number line once both are included
- a question asking which families a number belongs to is asking you to place it in this nesting

The decimal test

- a decimal that stops, such as 0.75, is rational
- a decimal that repeats a block for ever, such as 0.333... or 0.142857142857..., is also rational
- a decimal that is endless and never repeats is irrational, and that is the only irrational case
- so the test is simple: terminating or recurring means rational, neither means irrational
- a fraction $\frac{p}{q}$ in lowest terms terminates exactly when q has no prime factor other than 2 and 5
- so $\frac{3}{8}$ terminates because 8 is 2^3 , while $\frac{1}{6}$ recurs because 6 carries a factor of 3

the standard trick

Turning a recurring decimal into a fraction

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let x be the decimal, for instance $x = 0.363636...$

count the length of the repeating block, here two digits multiply by 10 raised to that length, so $100x = 36.363636...$

subtract the original from this: $100x - x = 36.363636... - 0.363636...$

the endless tails cancel exactly, leaving $99x = 36$

so $x = \frac{36}{99} = \frac{4}{11}$ after cancelling by 9