

primes are the building blocks, and some numbers refuse to be fractions

learn these

The Fundamental Theorem of Arithmetic

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every composite number $n = p_1 \times p_2 \times p_3 \times \dots \times p_k$

primes collected as powers $3825 = 3^2 \times 5^2 \times 17$

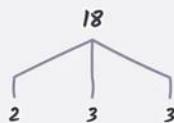
and 12, 18 the same way $12 = 2^2 \times 3, 18 = 2 \times 3^2$

the list of primes is fixed. Only the order can change, so two people always land on the same list

Turning a number into primes

- a prime has exactly two factors, 1 and itself. 1 is neither prime nor composite
- 2 is the only even prime, so every even composite carries at least one 2
- keep dividing by the smallest prime that fits: 2, then 3, then 5, then 7
- stop once the quotient is itself prime: $3825 \rightarrow 1275 \rightarrow 425 \rightarrow 85 \rightarrow 17$

Prime factors hanging from the number



12 hanging over the three primes it is built from: 2, 2 and 3

18 hanging over its own three primes: 2, 3 and 3

read the shared primes for the HCF (one 2 and one 3, so 6) and the fullest supply for the LCM (two 2s and two 3s, so 36)

HCF and LCM from the factorisation

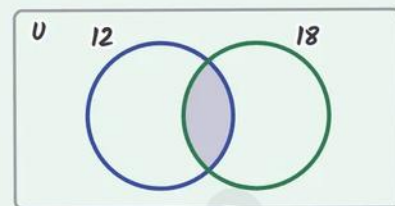
smallest power of each shared prime $HCF(12, 18) = 2 \times 3 = 6$

largest power of every prime seen $LCM(12, 18) = 2^2 \times 3^2 = 36$

the product law, two numbers only

$$HCF(a, b) \times LCM(a, b) = a \times b$$

check it $6 \times 36 = 216 = 12 \times 18$



the prime factors of 12 and of 18 as two circles: the overlap is the HCF's supply of primes, 2×3

the product law holds for exactly two numbers.

With three it breaks: $HCF(2, 3, 4) = 1$ and $LCM(2, 3, 4) = 12$, product 12, not 24

HCF and LCM: the special forms *

$$HCF(a, b) \times LCM(a, b) = a \times b$$

a and b share no factor, $HCF = 1$ → LCM is the plain product $LCM(a, b) = ab$

a divides b exactly → HCF is the smaller, LCM the larger $HCF = a, LCM = b$

$a = b$ → both collapse to the number itself $HCF = LCM = a$

for 2, 3 and 4 → product law fails $1 \times 12 = 12 \neq 24$

the first three rows all follow from the product law above. The last row is the limit it runs into, worked in full on the next page