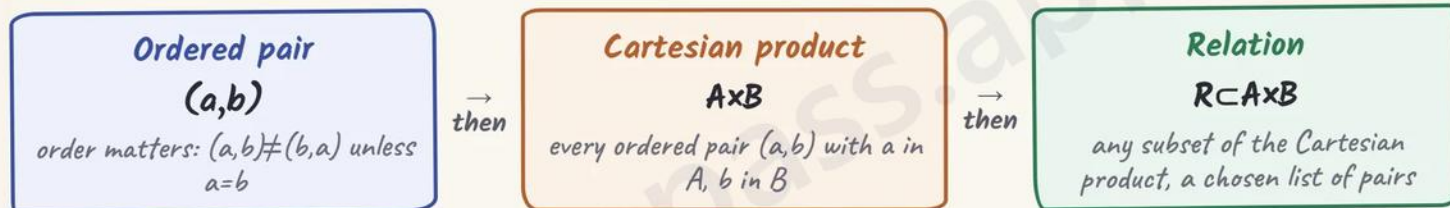


Relations and Functions

ordered pairs, grown into a rule between two sets



a relation is just a rule for WHICH pairs of $A \times B$ get picked, nothing more

Ordered pairs and the Cartesian product

equal exactly when both coordinates match $(a,b) = (c,d) \leftrightarrow a=c, b=d$

number of pairs in $A \times B$ $n(A \times B) = n(A) \times n(B)$

Cartesian product with itself $A \times A = \{(a,b) : a,b \in A\}$

$A \times B$ and $B \times A$ hold the same NUMBER of pairs but not the same pairs. Order flips every entry

a relation is just a subset of $A \times B$. A function is a relation that's extra fussy about the first coordinate, and that's the whole next page

A function is a fussy relation *

- every relation $R \subset A \times B$ qualifies, but a function additionally needs every element of A used exactly once
- no element of A may be left unused, and none may appear in two different pairs
- elements of B can repeat, or sit unused entirely: only A's side carries the strict rule
- written $f: A \rightarrow B$, and $f(a)=b$ means the pair (a,b) belongs to f

Counting relations and functions between finite sets

number of relations from A to B $2^{n(A)n(B)}$

number of functions from A to B $n(B)^{n(A)}$

a relation is just any subset of $A \times B$. This is "how many subsets does a set of this size have" again

Domain, co-domain, range

- domain: the set of first elements actually used, one from every pair in R
- co-domain: the WHOLE set B the relation maps into, not just the elements actually hit
- range: the set of second elements actually used, always a subset of the co-domain
- range = co-domain only when every element of B gets used at least once

Function notation

a function from A to B $f: A \rightarrow B$
 image of a under f $f(a)=b$
 domain, every valid input D_f
 range, every value actually output $R_f \subset B$