

Relations and Functions

which pairings survive being reflexive, symmetric, transitive, and which functions can be run backward

Relation
 $R \subset A \times A$
 any subset of ordered pairs from A to A

→
 add reflexive + symmetric + transitive

Equivalence relation
 R
 reflexive, symmetric AND transitive, all at once

→
 add reflexive + symmetric + transitive

Equivalence classes
 $[a]$
 A splits into disjoint pieces, every element in exactly one class

every equivalence relation comes from some partition of A , and every partition of A gives back an equivalence relation.
 The two ideas are really the same thing

Empty relation vs Universal relation

Empty relation

- R contains no pairs at all, $R = \emptyset$
- reflexive only if A itself is empty, otherwise it fails immediately
- vacuously symmetric and transitive, there's nothing to violate

Universal relation

- every pair (a,b) is in R , $R = A \times A$
- reflexive, symmetric, transitive all hold trivially
- always an equivalence relation, since the whole set collapses to one class

both ignore what the elements actually are. Check these two extremes first, then the interesting relations in between

Reflexive, symmetric, transitive: defined

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- reflexive: $(a,a) \in R$ for every $a \in A$, no element skipped
- symmetric: $(a,b) \in R \Rightarrow (b,a) \in R$, the pair runs both ways
- transitive: $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$, chains close up
- all three together, and only then, R earns the name equivalence relation

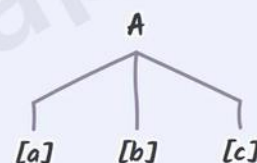
Equivalence classes

a always sits in its own class

$$a \in [a]$$

b belongs to a 's class exactly when they're related

$$b \in [a] \leftrightarrow aRb$$



A splits into disjoint equivalence classes, one branch per class

any two classes are either identical or share no elements. Together they partition all of A , nothing left over, nothing double-counted