

Relations and Functions

equivalence, one to one and onto, and the inverse that only exists sometimes

The three properties, stated so a proof can use them

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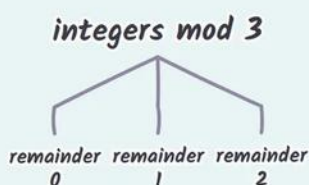
- **reflexive**: for EVERY a in A the pair (a, a) is in R , so a single missing self pair kills it
- **symmetric**: (a, b) in R forces (b, a) in R , for every such pair
- **transitive**: (a, b) and (b, c) in R force (a, c) in R
- **equivalence relation** is all three together, and it is what lets you treat different objects as the same for one purpose
- to prove one, take an **ARBITRARY** element or pair and argue in general; to disprove one, a single counterexample finishes the job

The relations to know as examples

- **equality** is the smallest equivalence relation on any set; the **universal relation** is the largest
- **parallelism of lines** is an equivalence relation, **perpendicularity** is not, since it fails both reflexivity and transitivity
- **similarity of triangles** is an equivalence relation and so is congruence; **being a divisor of** is reflexive and transitive but NOT symmetric
- **less than** on the reals is transitive only, and it is the standard example of a relation with exactly one of the three properties

Equivalence classes, and why they matter

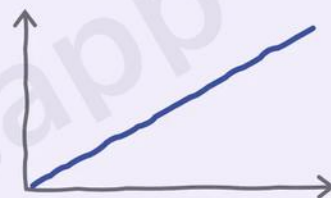
- the **class of a** is every element related to a , and two classes are either identical or completely disjoint
- so an equivalence relation **partitions** the set: every element lies in exactly one class and no class is empty
- on the integers, congruence modulo 3 gives exactly three classes, the remainders 0, 1 and 2
- a question asking for the number of elements in the class of a particular element is asking you to solve the relation's condition with that element fixed



one set partitioned into its equivalence classes: every integer lands in exactly one of the three

One to one, onto, and the pair of them

- **one to one (injective)**: different inputs have different images, so $f(x_1) = f(x_2)$ forces $x_1 = x_2$. That implication IS the proof method
- **onto (surjective)**: every element of the **CODOMAIN** is hit, so the range equals the codomain. The proof is to take an arbitrary y and produce an x that maps to it
- **bijective** is both, and only a bijection has an inverse
- **horizontal line test**: a horizontal line meeting the graph twice means not one to one
- for a differentiable function, **strictly increasing or strictly decreasing throughout** implies one to one, which is usually the quickest argument available



a strictly increasing straight line: no horizontal line meets it twice, which is one to one drawn