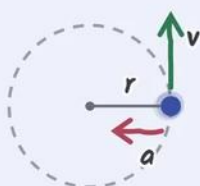


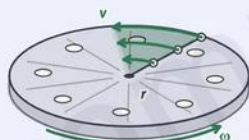


Circular motion, the kinematics

- revolution: axis outside the body; rotation: axis through it
- $\omega = \frac{d\theta}{dt}$ and $\alpha = \frac{d\omega}{dt}$, twins of v and a
- from angular to linear, θ in radians: $s = r\theta$, $v = \omega r$, $a_t = \alpha r$
- $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$; $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ lie along the axis, by the right hand thumb rule
- $\omega = 2\pi n = \frac{2\pi}{T}$, n in rev/s
- uniform circular motion: radial $a_r = \omega^2 r = \frac{v^2}{r} = v\omega$, toward the centre
- non-uniform adds a tangential $a_t = \alpha r$
- total $a = \sqrt{a_r^2 + a_t^2}$, the two being perpendicular
- $\boldsymbol{\alpha}$ points with $\boldsymbol{\omega}$ when speeding up, against it when slowing



v along the tangent,
radial acceleration at the
centre



the same angle at every
radius

Angular kinematic equations

average ω , constant α

$$\omega_{av} = \frac{\omega_0 + \omega}{2}$$

first equation

$$\omega = \omega_0 + \alpha t$$

second equation

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

third equation

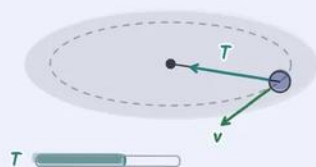
$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

only for constant α : the linear equations with s , u , v , a swapped for θ , ω_0 , ω , α

Centripetal and centrifugal force

Centripetal

smooth table

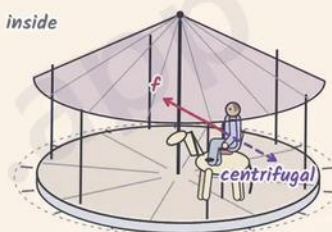


bob on a string: T alone is the centripetal force

- $\mathbf{F} = -m\omega^2 \mathbf{r}$, magnitude $\frac{mv^2}{r}$
- not a separate force: it names the resultant's direction
- written in the ground frame

Centrifugal

inside



merry-go-round, from inside: centrifugal balances F

- $\mathbf{F} = +m\omega^2 \mathbf{r}$, outward, felt in the rotating frame
- a pseudo force: the frame itself accelerates
- there, real forces plus it add to zero

centripetal force only keeps a body circling. Remove it: off along the tangent, not outward