

Sequences and Series

AP adds the same amount, GP multiplies the same factor, and HP is just a GP in disguise

Sequence, series, progression

- a sequence $t_1, t_2, t_3, \dots, t_n, \dots$ is numbers in a definite order, finite or infinite
- a series is what you get by adding the terms of a sequence: $t_1 + t_2 + t_3 + \dots$
- a sequence following a specific rule is a progression: AP, GP, HP, AGP here

AP, GP, HP at a glance

Progression	Rule	n th term	Sum of n terms
AP	constant difference d	$a + (n-1)d$	$\frac{n}{2} [2a + (n-1)d]$
GP	constant ratio r	ar^{n-1}	$\frac{a(r^n - 1)}{r - 1}, r \neq 1$
HP	reciprocals form an AP	no direct formula	flip to AP, solve, flip back

the same sequence can never be both AP and GP (unless it's constant), since one needs a fixed difference and the other a fixed ratio

Arithmetic Progression, a recap *

constant common difference $d = t_{n+1} - t_n$ $t_n = a + (n-1)d$

sum of n terms $S_n = \frac{n}{2} [2a + (n-1)d]$



AP terms plotted against n form a straight line: each step adds the same fixed d , so the growth rate never changes

adding or multiplying every term of an AP by a nonzero constant k still gives an AP: $t_1 + k, t_2 + k, \dots$ and $kt_1, kt_2, \dots$ are both AP

Geometric Progression

constant common ratio $r = \frac{t_{n+1}}{t_n}$ $t_n = ar^{n-1}$

a GP is written $a, ar, ar^2, ar^3, \dots$ Unlike d in an AP, r can never be zero, since every term would then collapse to 0 after the first