

Sequences and Series

an ordered list of numbers, and what happens when you add it up

Sequence vs series

- a sequence is an ordered list $a_1, a_2, a_3, \dots$, one number for every position n
- a series is what you get by adding the terms of a sequence together: $a_1 + a_2 + a_3 + \dots$
- a_n , the n th term, is usually given by a formula in n , so any term can be found without listing every one before it
- a sequence can be finite (it stops) or infinite (it never does)

Arithmetic progression (AP): a fixed amount added each time

n th term, first term a , common difference d

$$a_n = a + (n-1)d$$

common difference: any term minus the one before it

$$d = a_n - a_{n-1}$$

sum of the first n terms

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

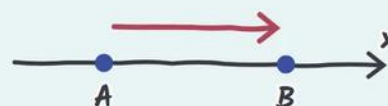
sum, written with the last term l instead of d

$$S_n = \frac{n}{2} (a + l)$$

d can be negative, the terms just count down instead of up. the formulas don't change either way

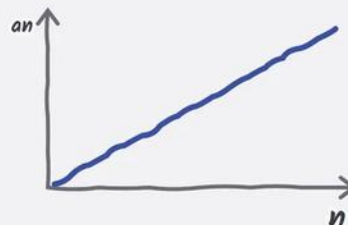
Arithmetic mean *

- the arithmetic mean of two numbers a and b is $A = \frac{a+b}{2}$, the number exactly halfway between them
- a, A, b is then a 3-term AP: $A - a = b - A$, the same common difference on both sides
- inserting n arithmetic means between a and b makes an AP of $n+2$ terms total, a and b included



two points A and B on the real line. their arithmetic mean sits exactly halfway between them

How AP terms grow *



terms of an AP plotted against n : a straight line, since each term adds the same fixed amount d to the last

Geometric progression (GP): a fixed ratio multiplied each time

n th term, first term a , common ratio r

$$a_n = a r^{n-1}$$

common ratio: any term divided by the one before it

$$r = \frac{a_n}{a_{n-1}}$$

sum of the first n terms, $r \neq 1$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

sum to infinity, only when $|r| < 1$

$$S_\infty = \frac{a}{1-r}$$

the infinite sum formula needs $|r| < 1$ to mean anything. for $|r| \geq 1$ the terms never shrink, so the sum never settles down