

a well-defined collection, the operations that combine two of them, and the ordered pairs that link them

What Makes a Set

- a set is a collection of WELL-DEFINED objects, called its elements or members
- well-defined means anyone can decide, without opinion, whether an object belongs
- 'days in a week' is well-defined, 'happy people in a town' is not
- $x \in A$ reads x belongs to A ; $x \notin A$ reads x does not belong to A
- three ways to describe a set: roster, set-builder, and a Venn diagram
- roster lists every element in braces; order and repeats don't matter
- set-builder states the defining property instead of listing members
- $n(A)$, the cardinality of A , is the count of distinct elements in a finite set A

Types of Sets

Empty set

no elements at all, written $\varnothing$ or $\{\}$; $n(\varnothing)=0$

Singleton

exactly one element, $n(A)=1$

Finite

a finite number of elements, including the empty set itself

Infinite

not finite: the natural numbers, or all points on a circle

Equal sets

$A=B$ exactly when $A \subseteq B$ and $B \subseteq A$: same elements, order and repeats irrelevant

Equivalent sets

$n(A)=n(B)$: same SIZE, not necessarily the same elements

Subset

$A \subseteq B$: every element of A lies in B

Proper subset

$A \subset B$: $A \subseteq B$, plus at least one element of B outside A

Power set $P(A)$

the set of all subsets of A , itself a set of sets. $n(A)=m$ gives $n(P(A))=2^m$

Universal Set

- every set in a discussion sits inside one fixed U : the universal set
- $\varnothing \subseteq$ every set, and every set $\subseteq$ itself