

a locus is a set of points obeying a rule, and a line is the simplest set whose slope never changes

Locus and the Equation of a Locus

- a locus is the set of all points satisfying one geometric condition, plural loci
- an equation OF a locus holds on every point of it, none off it
- method: let $P(x,y)$ be general, translate the condition into an x,y equation, simplify
- e.g. perpendicular bisector of AB: locus of P with $PA=PB$, a line
- e.g. bisector of an angle: locus of points equidistant from its arms, a ray

shifting the origin never bends the curve, it only relabels every point; whether a shape is a line, a circle or a parabola survives untouched

Inclination and Slope *

- inclination θ : smallest angle a line makes with the positive X-axis, anticlockwise; $0^\circ \leq \theta < 180^\circ$
- slope $m = \tan \theta$ when it exists; undefined for the Y-axis and lines parallel to it
- slope of the X-axis, and of lines parallel to it, is 0
- parallel lines share the same inclination, equivalently the same slope
- two points $A(x_1, y_1)$, $B(x_2, y_2)$ on a non-vertical line: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Q. Find the equation of the locus of points equidistant from $A(-3,0)$ and $B(3,0)$. Identify the locus.

- let $P(x,y)$ satisfy $PA=PB$, so $PA^2=PB^2$
- $(x+3)^2 + y^2 = (x-3)^2 + y^2$
- $x^2 + 6x + 9 = x^2 - 6x + 9$, so $12x = 0$

Answer

$x=0$: the locus is the Y-axis, the perpendicular bisector of AB

Perpendicular and Angle Between Lines

perpendicular

$$m_1 m_2 = -1$$

acute angle between two lines

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

assumes the lines are not perpendicular, since $m_1 m_2 = -1$ would divide by zero

Shift of Origin

- shift the origin to $O'(h,k)$, axes kept parallel: $x = x' + h$, $y = y' + k$
- to move a locus into the new axes, substitute $x' + h$ for x , $y' + k$ for y , simplify

Q. Find the acute angle between lines of slope 3 and -2

$$\tan \theta = \left| \frac{3 - (-2)}{1 + (3)(-2)} \right| = \left| \frac{5}{-5} \right| = 1$$

Answer $\tan \theta = 1$, so $\theta = 45^\circ$

Q. The origin is shifted to $(2,3)$, axes parallel. Find the new equation of $x^2 - xy - 2y^2 - x + 4y + 2 = 0$

- substitute $x = X + 2$, $y = Y + 3$
- $(X+2)^2 - (X+2)(Y+3) - 2(Y+3)^2 - (X+2) + 4(Y+3) + 2 = 0$
- expand and collect every constant and cross term

Answer $X^2 - XY - 2Y^2 - 10Y - 8 = 0$