

a line needs a direction and a point, a plane needs a normal and a distance

Direction cosines vs direction ratios

Direction cosines (l, m, n)

- cosines of the angles the line makes with the x, y, z axes
- always satisfy $l^2 + m^2 + n^2 = 1$
- unique for a given directed line, reversing the direction flips every sign

Direction ratios (a, b, c)

- any numbers proportional to l, m, n
- not unique: (2,4,6) and (1,2,3) describe the same direction
- recovered as $l = a / \sqrt{a^2 + b^2 + c^2}$, and likewise for m and n

ratios are the everyday working numbers. Cosines are what you get after normalising them, and $l^2 + m^2 + n^2 = 1$ is the one check that confirms it's actually been done

Equation of a line

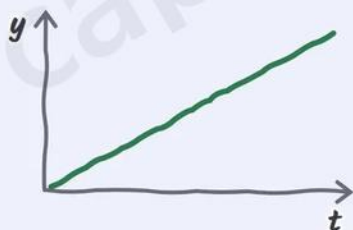
vector form, through a point a, direction b $r = a + \lambda b$

cartesian form, through (x_1, y_1, z_1) , direction ratios a,b,c

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

through two points

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$



each coordinate of a point on a line changes at a constant rate as the parameter t increases: exactly why x, y, z are linear in t along a straight line

Angle between two lines *

direction

ratios

(a_1, b_1, c_1) and (a_2, b_2, c_2)

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

perpendicular exactly when

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

parallel exactly when

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

perpendicular is a dot-product test on the direction ratios. Parallel is a proportion test on the same numbers. The two conditions never overlap for a genuine pair of distinct directions

Two lines, three possible relationships



two lines that intersect: their point of meeting is common to both, and everything before or after it belongs to only one line

two segments running the same direction: proportional direction ratios, the same idea behind why parallel lines never meet

a third possibility exists only in three dimensions: two lines that are neither parallel nor ever meet, called skew

direction ratios are the working numbers, and everything else, cosines, angle, parallel, perpendicular, is computed FROM them