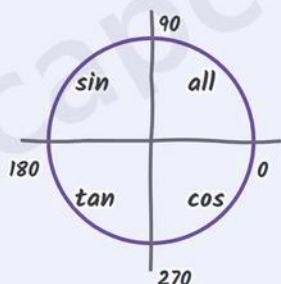


equations that repeat forever, triangles solved from one side, and functions run backward

Trigonometric Equations *

- an equation in trigonometric ratios of an unknown angle: e.g. $\sin\theta = \frac{1}{2}$, $\tan\theta = 2$, $\cos 3\theta = \cos 5\theta$
- any value of θ satisfying the equation is a solution
- every trig ratio repeats (sin, cos every 2π , tan every π), so infinitely many solutions can exist
- some equations have no solution at all: $\sin\theta = 3$ is impossible, since $\sin\theta$ never leaves $[-1, 1]$



the sign picture behind every trig equation: all three ratios positive in quadrant 1, only sin in 2, only tan in 3, only cos in 4

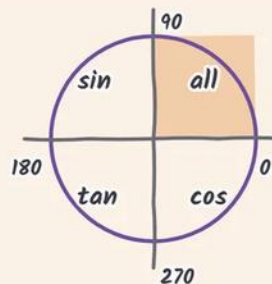
Principal Solutions *

the interval searched, one full revolution

$$0 \leq \theta < 2\pi$$

worked: $\cos\theta = \frac{1}{2}$ has two principal solutions

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$



the first principal solution, $\pi/3$, sits in quadrant 1, where cosine is positive

a principal solution is just any solution that lands inside one full revolution, $[0, 2\pi)$. Past that window the same angle keeps recurring, which is what the general solution captures

General Solution

$$\sin\theta = \sin\alpha \quad \theta = n\pi + (-1)^n\alpha$$

$$\cos\theta = \cos\alpha \quad \theta = 2n\pi \pm \alpha$$

$$\tan\theta = \tan\alpha \quad \theta = n\pi + \alpha$$

$$\sin\theta = 0 \quad \theta = n\pi$$

$$\cos\theta = 0 \quad \theta = (2n+1)\frac{\pi}{2}$$

$$\tan\theta = 0 \quad \theta = n\pi$$

$$\text{squared sin, cos or tan equation} \quad \theta = n\pi \pm \alpha$$

n stands for any integer. Match the FORM first, sin with sin, cos with cos, tan with tan, before picking a template. Squaring loses the sign, so the squared equation combines $\theta = n\pi + \alpha$ and $n\pi - \alpha$ into $n\pi \pm \alpha$. Verify with $n=0, 1$ directly