

the unit circle's ratios, and every identity built from them

Degrees and radians *

one full revolution

$$360^\circ = 2\pi \text{ rad}$$

straight angle

$$180^\circ = \pi \text{ rad}$$

degrees to radians

$$\theta_{\text{rad}} = \theta_{\text{deg}} \times \frac{\pi}{180}$$

π radians is a straight line, not a full circle. A common mix-up is doubling it by mistake

Reciprocal and quotient identities

reciprocal of sin

$$\csc \theta = \frac{1}{\sin \theta}$$

reciprocal of cos

$$\sec \theta = \frac{1}{\cos \theta}$$

reciprocal of tan

$$\cot \theta = \frac{1}{\tan \theta}$$

tan as a quotient

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Signs by quadrant: All Sin Tan Cos

Quadrant I, 0° to 90°

all six ratios **positive**

Quadrant II, 90° to 180°

only sin and cosec **positive**,
rest negative

Quadrant III, 180° to 270°

only tan and cot **positive**,
rest negative

Quadrant IV, 270° to 360°

only cos and sec **positive**,
rest negative

Domain and range

- sin θ and cos θ : domain is every real θ , range is $[-1, 1]$
- tan θ and sec θ : undefined wherever cos $\theta = 0$, that is $\theta = (2n+1)\frac{\pi}{2}$
- cot θ and cosec θ : undefined wherever sin $\theta = 0$, that is $\theta = n\pi$
- tan θ and cot θ : range is all of $\mathbb{R}$, with no bound at all
- sec θ and cosec θ : range skips $(-1, 1)$ entirely, never a value strictly between -1 and 1

every ratio is read straight off the unit circle: cos θ is the x-coordinate, sin θ is the y-coordinate, **nothing more**

The three Pythagorean identities

the basic one

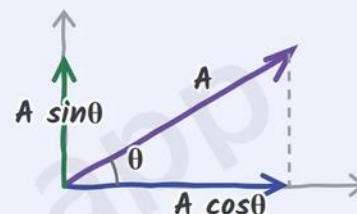
$$\sin^2 \theta + \cos^2 \theta = 1$$

divide through by $\cos^2 \theta$

$$1 + \tan^2 \theta = \sec^2 \theta$$

divide through by $\sin^2 \theta$

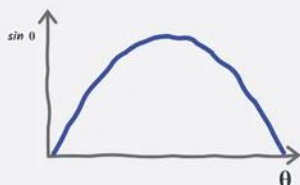
$$1 + \cot^2 \theta = \csc^2 \theta$$



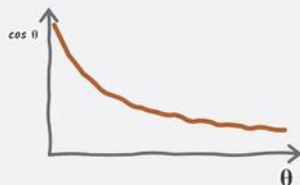
a length A at angle θ : its two sides are $A \cos \theta$ and $A \sin \theta$, so their squares add back to A^2

all three are the SAME identity, just divided through by a different square, so derive the last two instead of memorising them cold

How sin and cos move *



sin θ from 0 to π : rises to 1 at $\pi/2$, back down to 0



cos θ : starts at 1 , falls as θ grows

sin θ is an odd function, cos θ is even: $\sin(-\theta) = -\sin \theta$ but $\cos(-\theta) = \cos \theta$