



Trigonometry - 2

sums, differences, doubles and triples of angles, folded back into single ratios

Compound Angles

- a compound angle : $A+B$ or $A-B$, treated as a single angle
- $\cos(A-B) = \cos A \cos B + \sin A \sin B$
- $\cos(A+B) = \cos A \cos B - \sin A \sin B$
- get $\cos(A+B)$ from $\cos(A-B)$ by putting $-B$ in place of B , since $\cos(-B) = \cos B$ and $\sin(-B) = -\sin B$

unit circle chord PQ, worked two ways

Where $\cos(A-B)$ comes from

on the unit circle put P at angle A and Q at angle B:

$P(\cos A, \sin A)$, $Q(\cos B, \sin B)$

squared chord length by the distance formula:

$$[d(PQ)]^2 = 2 - 2(\cos A \cos B + \sin A \sin B)$$

now measure the SAME chord after turning the axis so OQ becomes the new x-axis: the angle between the rays is $A-B$, so P becomes $(\cos(A-B), \sin(A-B))$ and Q becomes $(1, 0)$

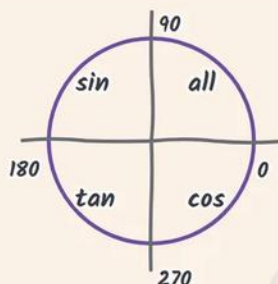
that gives the same chord as $[d(PQ)]^2 = 2 - 2\cos(A-B)$

a chord's length does not depend on where the axes point, so the two expressions are equal:

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

Sum and Difference Formulae *

sine	$\sin(A+B) = \sin A \cos B + \cos A \sin B$
sine	$\sin(A-B) = \sin A \cos B - \cos A \sin B$
tangent	$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
tangent	$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
cotangent	$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$
cotangent	$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$



the four quadrants: whichever ratio of A , B or $A \pm B$ a step needs, its sign still comes from here

$\sin(A+B)$ comes from $\cos(\pi/2 - (A+B)) = \cos((\pi/2 - A) - B)$ expanded the same way; $\tan$, $\cot$ follow by dividing the sine and cosine expansions through. both $\cot$ forms need A , B and $A \pm B$ to not be multiples of π

