



magnitude and direction together, added tip to tail, multiplied two different ways

## Scalars, Vectors, and the Types That Matter

- a scalar needs only a magnitude, e.g. mass, length, time
- a vector needs BOTH magnitude and direction, e.g. displacement, velocity, force
- a vector  $\overrightarrow{AB}$  is a directed segment: A the initial point, B the terminal point
- its magnitude is  $|\overrightarrow{AB}| = l(\overrightarrow{AB})$ , never negative
- the zero vector  $\mathbf{0}$  has zero magnitude and no definite direction
- a unit vector has magnitude exactly 1, written  $\hat{a}$  for the unit vector along  $\mathbf{a}$
- collinear vectors share a line of action, whatever their length or sense
- coplanar vectors lie in the same or in parallel planes
- every vector here is treated as FREE: it can be translated anywhere in space
- without changing its magnitude or direction

## Four vectors that are easy to mix up

### Equal vectors

same magnitude AND same direction; positions can differ entirely

### Negative of a vector

same magnitude, OPPOSITE direction: if

$\overrightarrow{PQ}$

=

$\mathbf{a}$

then

$\overrightarrow{QP}$

= -

$\mathbf{a}$

### Collinear vectors

same line of action only; can differ in magnitude or point opposite ways

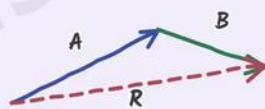
### Co-initial vectors

share one starting point but generally point in different directions

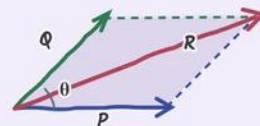
equal is the strict case: same magnitude and direction together. Collinear only needs the same line, nothing about length or sense

## Adding two vectors, two pictures of the same sum

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triangle law: place the tail of B at the tip of A, and the resultant R closes the triangle straight back to the start



parallelogram law: P and Q drawn from a common point, the diagonal through that point is their sum

same two vectors, same resultant, either construction: use the triangle law when the vectors act one after another, the parallelogram law when they act at the same instant

## Laws of Vector Addition

commutative

$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$$

associative

$$(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$$

additive identity and inverse

$$\mathbf{a} + \mathbf{0} = \mathbf{a}, \mathbf{a} + (-\mathbf{a}) = \mathbf{0}$$

triangle inequality

$$|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$$

subtraction is addition of the negative:  $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$ .  
Scalar multiplication  $k\mathbf{a}$  scales the magnitude to  $|k||\mathbf{a}|$  and flips direction exactly when  $k < 0$

if three vectors along the sides of a triangle are traversed in ORDER, tail to tip all the way round, their sum is  $\mathbf{0}$ : the path closes on itself and returns to where it started